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REVIEW

Hydrodynamics of Suspensions. By M. UNGARISH. Springer, 1993. 322 pp. DM 138.

About a decade and a half ago, research on fluid mechanical aspects of settling suspensions entered a phase of significant growth. Several seminal contributions that were published during a couple of years opened up new routes to successful research. As the field is one of both scientific and technological importance, it is certainly a good idea to sum up what has been done. This intention is well fulfilled by the book by Professor M. Ungarish, who has himself made several important contributions to this subject.

One of the main themes of the book is to clarify the sometimes very profound differences between settling due to gravity and that due to a centrifugal force field. The primary reason for these differences is that in most settling devices used in industry, the rather passive settling process itself is accompanied by a buoyancy-driven motion due to the difference in specific weight between suspension and clear fluid. Thus, effects of the Coriolis force will almost always lead to completely different behaviour of settling suspensions in non-rotating and rotating vessels. It may be worth pointing out that the main title of the book is slightly misleading. In its present form, the title does not indicate that by far the most thoroughly discussed class of phenomena is related to settling. This may give the impression that, for example, mechanical properties of suspensions, which is probably an even larger field than that discussed in the book, are also taken up. In the book, the latter aspect of the hydrodynamics of suspensions is discussed very briefly, which *per se*, is not a shortcoming.

The book is well written and edited. The style is vivid and enthusiastic. The mathematics is presented in a consistent way and usually supplemented by pedagogically well formulated explanations in mechanical terms. The level aimed at is high and the author has taken great care to not only derive but also explain, in detail, many of the seminal contributions to the subject. However, even though this is all well done, it makes the book a bit heavy to read. As an example, in several chapters, certain problems are examined by using two different models for the mechanical properties of the suspension. In all cases, the results are very similar. Detailed analysis using one of the models only and a brief summary of results from the other one would, in the reviewer's opinion, have been sufficient.

Even though most of the contributions to the field that are dealt with in the book are theoretical results, there are some experimental investigations reported on in the literature as well. Some of these experimental results are presented in the text, including some interesting photographs. But the inclusion of more experimental results would have given the book a better balance between theory and experimental verification. After reading this book, the present reviewer gets the impression that the theory of settling two-phase flows lacks experimental verification to an extent that is not always admitted.

A short but illustrative account of various general aspects of settling is given in chapter 1. Theoretical arguments are supported by numerical examples.

Chapter 2, which is entitled 'Physico-mathematical formulation', deals with the continuum modelling of suspensions. Basic concepts, such as averages and kinematic shocks, are introduced. Commonly used estimates for the viscosity of suspensions are discussed in some detail. A difficult issue in the hydrodynamics of suspensions is the

formulation of equations for the balance of momentum of the suspension. There is a large literature on this aspect of the subject, pertinent parts of which are briefly summarized in the text. Two of the most frequently used models, the 'diffusion' or 'mixture' model and the somewhat more complicated 'two-fluid' model, are presented and discussed in some detail. Both these models are used extensively later in the book. There is considerable discussion on the art of choosing reasonable estimates for unknown functions that quantify interaction between particles and fluid. On the whole, this is well done even though the present reviewer on occasion gets the feeling that the author of the book sometimes places more credibility in the models than they deserve. This matter is clearly demonstrated by the discussion in subsection 5.4 in the book, about which more will be said below.

A well-written summary of basic results in the theory of rotating fluids is given in chapter 3. The reader is provided with mathematically simple but mechanistically transparent explanations of the flow in Ekman, von Kármán and Stewartson layers, the role of geostrophic contours, etc. The final section of this chapter is a review of the so far not altogether coherent theoretical and experimental results for forces on particles in a rotating fluid that are given in the literature.

The text in chapter 4 is devoted to a set of idealized problems where effects of wall friction are neglected. The only role of walls here is to support the formation of layers of sediment and clear fluid. The chapter deals with settling of an initially homogeneous suspension and is split into two parts. Gravity-driven settling between two large horizontal walls is taken up in the first part. Effects of viscosity would here be confined to regions close to vertical walls that are assumed to be located far away from the region considered. In the second part of the chapter, centrifugal settling between very long rotating concentric cylinders with slippery walls is considered. For the case of gravity-driven settling, some very reasonable assumptions lead to identical solutions for both the mixture and two-fluid models. For centrifugal settling, a similarity solution is computed numerically for the two-fluid model. This solution is supplemented with several perturbation solutions. The presentation in this chapter is mathematically simple but a bit long. Some comments on effects of polydispersivity are also given.

The theme of chapter 5 is effects of wall friction on settling of an initially rigidly rotating homogeneous suspension. The chapter begins with an interesting exposition of the linear spin-up problem of a dilute suspension. It is clearly demonstrated, e.g. for settling of heavy particles, how the spin-down due to radial transport of heavy particles, which tend to conserve angular momentum, competes with spin-up due to Ekman suction. A subsection is devoted to an investigation of the $E^{1/4}$-layer. Also spin-up from rest is considered using a Wedemeyer-type empirical formula for the Ekman suction. Using some interesting heuristic arguments, especially about the role of Ekman layer transport, a reasonable solution, which agrees qualitatively with experiments by the author, is found. The chapter ends with a thorough description of an attempt to compute the counterpart of the classical von Kármán solution for a rotating disc. The computations point at serious weaknesses in the models used regarding the description of particle motion near solid walls. One of these deficiencies is that the solution of the model equations has boundary-layer behaviour on the lengthscale given by the particle size, which is inconsistent with the continuum model of the suspension.

Chapter 6 is a detailed treatment of settling due to gravity in vessels with inclined walls. The text starts with a well-phrased explanation of the purely kinematic Ponder–Nakamura–Kuroda theory for enhanced settling in tilted vessels, the so-called

Boycott effect. Thereafter, several cases that have been investigated by asymptotic methods by different authors are presented in a systematic way. 'Wide' and 'narrow' containers are dealt with separately. In the former case, a full account is given of the different types of boundary layers, whose structures depend on the different orders of magnitude of the Grashof and Reynolds numbers, that appear at solid walls. A mathematical controversy in this field, pointed out by Amberg *et al.* (*J. Fluid Mech.* vol. 165, 1986, p. 473) but not touched upon in this chapter, is the presence of so-called sonic shocks on the interface between suspension and clear fluid in narrow channels. The chapter ends with a short discussion of available experimental results and questions of stability.

In chapter 7, a variety of intriguing phenomena that appear in centrifugal settling are taken up for detailed discussion with emphasis put on the often unexpected differences compared with settling due to gravity in seemingly similar vessels. Among other things, the removal by rotation, due to the appearance of a swirling flow, of the Boycott effect in an axisymmetric container and its restoration, albeit in modified form, by insertion of a radial barrier is explained. Another topic, which is important especially from a technological point of view, is settling between narrowly spaced rotating conical plates. Also, different aspects of the formation of clear fluid layers on meridional walls due to differential circumferential motion of particles and fluid, driven by the Coriolis force, are penetrated at some depth in this chapter. Although the theory of settling suspensions frequently makes use of characteristics, there are, surprisingly, no diagrams showing characteristics in the present book. In the earlier chapters this may not matter very much but the presentations in chapters 6 and 7 would have gained by including some graphical illustrations of shock paths and characteristics.

The last chapter deals with numerical solution of some idealized test problems using both the two-fluid model and the mixture model. Comparisons are made with asymptotic solutions and it is found that, even for rather coarse meshes, agreement with numerical solutions is quite good. This consideration of computational aspects of the subject is, in this reviewer's opinion, very rewarding. This is certainly a virgin field of great technological importance.

Chapters 2–8 end with a set of problems. Most of these are of a theoretical nature and deal with clarifications and extensions of issues taken up in the main text. Obviously, the author has taken trouble over the formulation of these problems and they do certainly provide the reader with a deeper understanding of the subject. However, a larger fraction of conceptually simple problems of an exploratory or a numerical nature illustrating, for example, orders of magnitude in simplified cases would possibly have made an improvement.

To sum up, Ungarish's book on settling is a good one. For somebody who intends to take up studies in the field, either in university research or in industrial development, it will serve as a updated thorough encyclopaedia of the subject. As always, there are a few things, some of which have been briefly discussed above, that a reviewer would like to see done in a different way. In the present case, however, these minor reservations do not notably diminish a good general impression of the book, which is thus recommended.

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